1. **Small network structure**

Draw the reaction system

\[ P_1 + S_2 \leftrightarrow S_1 + S_3 \]
\[ S_1 + S_3 \leftrightarrow P_2 + S_2 \]

as a network, write down its stoichiometric matrix, and determine potential fluxes in stationary state.

Are there linear conservation relations? (\(v_i\): reaction rate, \(S_j\): internal (variable) compounds, \(P_k\): external (fixed) compounds)

2. **From the ODEs back to the reaction**

Consider the following system of differential equations. Try to find out a reaction scheme that corresponds to these reactions. Are there cases of activation or inhibition?

\[
\frac{dM_1^-}{dt} = -k_1 \cdot M_1^- \cdot \frac{1}{(1 + M_3^+)} + k_2 \cdot M_1^+ \\
\frac{dM_1^+}{dt} = +k_1 \cdot M_1^- \cdot \frac{1}{(1 + M_3^+)} - k_2 \cdot M_1^+ \\
\frac{dM_2^-}{dt} = -k_3 \cdot M_2^- \cdot M_1^+ + k_4 \cdot M_2^+ \\
\frac{dM_2^+}{dt} = +k_3 \cdot M_2^- \cdot M_1^+ - k_4 \cdot M_2^+ \\
\frac{dM_3^-}{dt} = -k_5 \cdot M_3^- \cdot M_2^+ + k_6 \cdot M_3^+ \\
\frac{dM_3^+}{dt} = +k_5 \cdot M_3^- \cdot M_2^+ - k_6 \cdot M_3^+
\]

3. **The repressilator** The repressilator (M. Elowitz and S. Leibler, Nature 2000) is a genetic circuit consisting of three proteins, each inhibiting the production of the following protein in a circle. Consider the following kinetic equations for synthesis and degradation of the proteins:

\[
v_i^{\text{syn}} = \frac{\beta}{1 + x_l(i)/k} \\
v_i^{\text{degr}} = \alpha \cdot x_i
\]

with \(i = 1, 2, 3\) and \(l(1) = 3, l(2) = 1, l(3) = 2\).

The reaction rate vector reads \(v = (v_1^{\text{syn}}, v_2^{\text{syn}}, v_3^{\text{syn}}, v_1^{\text{degr}}, v_2^{\text{degr}}, v_3^{\text{degr}})^T\).

(a) Write down the stoichiometric matrix and the differential equations for the protein levels \(x_i\).

(b) Consider a stationary state with \(x_1 = x_2 = x_3 = \bar{x}\). Calculate the elasticity matrix \(\varepsilon^S\). (Hint: it is not necessary to compute the value of \(\bar{x}\).)

(c) Compute the Jacobian matrix by the formula \(M = N\varepsilon^S\).

(d) How could you decide based on \(M\) whether the stationary state is stable?